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SKIN FRICTION AND HEAT TRANSFER FOR LIQUID FLOW OVER A POROUS WALL WITH GAS INJECTION*

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Abstract—Some theoretical calculations have been performed to determine the effect of bubbling by gas injection through a porous wall on the skin friction and heat transfer characteristics in the boundary layer of a liquid flowing over it. The results show a significant increase in both the skin friction and the heat transfer coefficient.

Some calculations for a case in which the injected gas makes a continuous film over the surface are also given. In this case, the skin friction and heat transfer drastically decrease.

NOMENCLATURE

- B, parameter defined by equation (17);
- C, parameter defined by equation (16);
- C_D, drag coefficient of gas bubble moving in liquid;
- c_p, specific heat;
- D, bubble diameter at detachment from plate surface;
- D', bubble diameter while attached to the surface;
- D_m, time-averaged bubble diameter defined by equation (4);
- d, equivalent pore diameter;
- d', diameter of sintered spheres which compose porous material;
- f_w, local skin friction coefficient on a flat plate with gas injection;
- f_{w_0}, local skin friction coefficient on a flat plate without gas injection;
- f_{w_m}, total skin friction coefficient on a flat plate with gas injection;
- g, gravitational acceleration;
- g_c, conversion factor between force and mass;
- h, heat transfer coefficient;
- k, thermal conductivity;
- k_w, surface roughness of rough plate;
- l, length;
- N, number of pores per unit surface area of porous plate;
- n, constant introduced in equation (9);
- Nu_x, local Nusselt Number with gas injection;
- Nu_{x_0}, local Nusselt Number without gas injection;
- Pr, Prandtl Number;
- Q_G, volume flow of gas injected into liquid flow per unit time and unit surface area (uniform for the entire surface);
- q, heat flux;
- R, volume fraction;
- Re_x, dimensionless group $\equiv \dfrac{x u_{L_\infty}}{\nu_L}$;
- Re_δ, dimensionless group $\equiv \dfrac{\delta u_{L_\infty}}{\nu_L}$;
- St, Stanton Number $\equiv \dfrac{h}{c_{p_L} \rho_L u_{L_\infty}}$;
- T, temperature;
- t, time;
- Δt, time required for bubble growing while attached to the surface;
- u, velocity component parallel to plate surface;
- V, bubble volume;
- v, velocity component perpendicular to plate surface;

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x, co-ordinate parallel to surface measured from leading edge of plate;
x_0, length of non-porous section at leading edge (no injection);
y, co-ordinate perpendicular to plate measured from surface.

Greek symbols

α, slip ratio $\equiv \frac{u_G}{u_L}$;
δ, boundary layer thickness;
ρ, density;
μ, dynamic viscosity;
ν, kinematic viscosity;
σ, surface tension at liquid–gas interface;
τ_w, shearing stress at plate surface;
ξ, velocity ratio $\equiv \frac{u_{L_G}}{u_{L_\infty}}$.

Subscripts

∞, value in free stream;
G, value in gas phase;
L, value in liquid phase;
m, averaged value;
w, value at plate surface;
x, value at x.

INTRODUCTION

THE DRAG and the heat transfer coefficient between a surface and a fluid flowing over it are remarkably affected by changing the structure of the momentum and the thermal boundary layer, respectively. Nucleate boiling is an example of increasing the heat transfer from a solid to a liquid. This can be explained by agitation of the boundary layer caused by the generation and the motion of the gas bubbles. As is well known, the main resistance to heat transfer is restricted to a thin region close to the wall, the thermal boundary layer, so that any mechanical agitation in this region should considerably change the rate of heat transfer. It seems reasonable, therefore, to expect that if a small amount of gas were injected through a porous surface into a liquid flowing over it and bubbles were formed on the surface, the liquid heat transfer coefficient would significantly increase. There is only one former experimental work by Gose *et al.* [1] based on such an idea. The results obtained show a significant increase in the heat transfer coefficient by injecting a small amount of gas. In some circumstances, the gas bubbles injected from the porous plate into a liquid flowing over it coagulate and make a stable and continuous film of gas over the surface. In such a case, the heat transfer and the friction drag are considerably reduced by the insulating effect of the gas layer.

The purpose of the present investigation is to study analytically the effect of gas injection through a porous flat plate on the skin friction drag and the heat transfer characteristics in the boundary layer of a liquid flowing over the surface.

1. CALCULATION ON A MIXED TWO-PHASE BOUNDARY LAYER. ANALYSIS

The first physical situation which has been chosen for study here is the forced convection, turbulent boundary layer flow over a flat porous plate as shown in Fig. 1. A stream of pure liquid approaches the plate. A gas is forced through

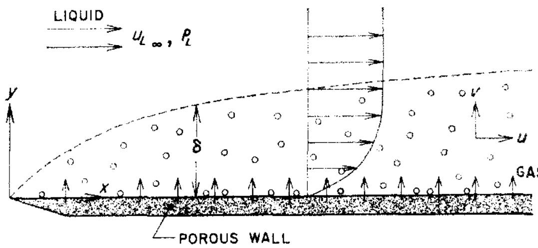

FIG. 1. Physical model and co-ordinate system.

the porous plate and injected into the boundary layer. Gas bubbles are formed in the boundary layer. Both the liquid and gas phase coexist in the boundary layer, and it is assumed that the gas phase does not exist beyond the boundary layer and that the gas phase is non-condensing. The effect of evaporation at the gas–liquid interface is neglected. The effect of the gravity force is also ignored.

The control surface is taken in the flow as shown in Fig. 2.

The momentum flow through the plate 1–2 is $\int_0^l (R_G \rho_G u_G^2 + R_L \rho_L u_L^2)\, dy$, where R denotes the volume fraction, and the subscripts G and L indicate the gas and the liquid phase, respectively.

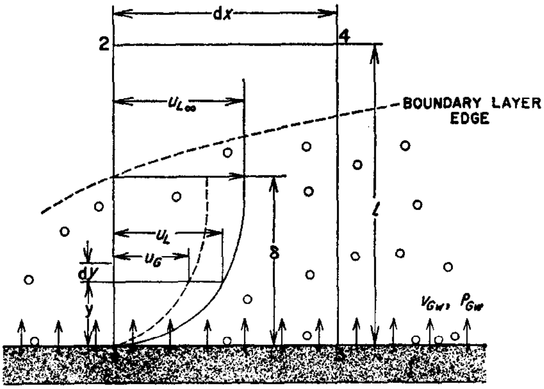

FIG. 2. Calculation of the flow boundary layer.

In progressing by the small distance dx, this momentum flow changes by

$$\mathrm{d}x \frac{\mathrm{d}}{\mathrm{d}x} \int_0^l (R_G \rho_G u_G^2 + R_L \rho_L u_L^2)\, \mathrm{d}y.$$

Considering the volume flow of gas injection through the wall 1–3, $Q_G \mathrm{d}x$, the volume flow through the plane 2–4 is

$$\mathrm{d}x \frac{\mathrm{d}}{\mathrm{d}x} \int_0^l (R_G u_G + R_L u_L)\, \mathrm{d}y - Q_G\, \mathrm{d}x.$$

This flow through the plane 2–4 is the liquid phase only, and it has the velocity component of $u_{L\infty}$ in the x-direction. Therefore, the x-momentum flow through the plane 2–4 is

$$u_{L\infty} \rho_L \left[\mathrm{d}x \frac{\mathrm{d}}{\mathrm{d}x} \int_0^l (R_G u_G + R_L u_L)\, \mathrm{d}y - Q_G\, \mathrm{d}x \right].$$

By neglecting the pressure gradient along the wall, the whole increase in x-momentum is equated with the shear stress τ_w along the wall 1–3 as follows:

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_0^\delta [R_G u_G (\rho_L u_{L\infty} - \rho_G u_G) + R_L \rho_L u_L (u_{L\infty} - u_L)]\, \mathrm{d}y - u_{L\infty} Q_G \rho_L = \tau_w. \tag{1}$$

The integration limit has now been changed since, in the range $\delta < y < l$, $R_G = 0$ and $(u_{L\infty} - u_L) = 0$ from the assumptions.

For the calculation, it is assumed that the velocity profile of the liquid is given by Prandtl's equation

$$\frac{u_L}{u_{L\infty}} = \left(\frac{y}{\delta}\right)^{1/7} \tag{2}$$

and that the shearing stress at the wall is given by the rough plate equation [2]

$$\tau_w = \tfrac{1}{2} \rho_L u_{L\infty}^2 \left[2{\cdot}87 + 1{\cdot}58 \ln \frac{x}{k_w} \right]^{-2\cdot5} \tag{3}$$

where the surface roughness k_w is originally introduced by Nikuradse as the grain size of the sand glued on the wall surface as tightly as possible. It is considered here that the injected bubbles act as surface roughnesses hydrodynamically and the grain size k_w is replaced by some characteristic length referred to the bubble diameter. The bubbles grow up on the surface, detach and new bubbles grow again, so that the system is not steady state. For simplicity, the time-averaged diameter of the growing bubble is taken here for replacing k_w:

$$D_m = \frac{\int_0^{\Delta t} D'\, \mathrm{d}t}{\int_0^{\Delta t} \mathrm{d}t}$$

where Δt is the time required for bubble growing while attached to the surface. The bubble growth rate can be equated to the volume flow of the injecting gas, Q_G, and the number of bubbles per unit area of the surface as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\pi}{6} D'^3\right) = \frac{Q_G}{N}$$

where the bubble is assumed to keep the spherical shape. Therefore,

$$\frac{\mathrm{d}D'}{\mathrm{d}t} = \frac{Q_G}{N} \cdot \frac{2}{\pi D'^2}.$$

Introducing this equation into the equation of D_m gives

$$D_m = \frac{\int_0^D \frac{N}{Q_G} \cdot \frac{\pi D'^3}{2}\, \mathrm{d}D'}{\int_0^D \frac{N}{Q_G} \cdot \frac{\pi D'^2}{2}\, \mathrm{d}D'} = \tfrac{3}{4} D \tag{4}$$

where D is the bubble diameter at detachment from the surface. The bubble diameter at detachment from the surface D can be determined from a balance of the fluid force acting on the bubble

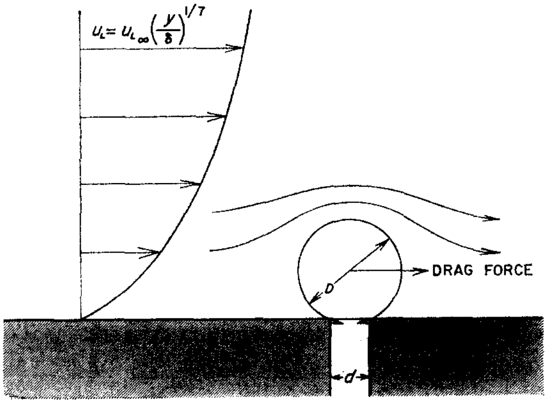

FIG. 3. Calculation of the bubble diameter.

and the surface tension force (Fig. 3). The drag force acting on the bubble is

$$C_D \cdot \rho_L \cdot \frac{\pi}{4} D^2 \frac{\bar{u}_L^2}{2g_c}$$

where C_D is the drag coefficient and $\bar{u}_L$ is the effective fluid velocity to the bubble which is approximated as a mean velocity:

$$\bar{u}_L = \tfrac{1}{2} u_{L\infty} \left(\frac{D}{\delta}\right)^{1/7}.$$

The surface tension force at the pore is a little complicated. Generally three surface tensions are acting, namely, surface tensions acting on the liquid–gas, liquid–solid, and gas–solid interface. Furthermore, the bubble actually changes its shape from sphere. For simplicity, the order of magnitude of the surface tension balanced with the above mentioned drag force can be considered as $\pi\sigma d$, where σ is the surface tension of liquid–gas interface and d is the equivalent pore diameter.

By equating the forces due to drag and surface tension,

$$\pi\sigma d = C_D \cdot \rho_L \cdot \frac{\pi}{4} D^2 \cdot \frac{1}{2g_c} \left[\frac{u_{L\infty}}{2}\left(\frac{D}{\delta}\right)^{1/7}\right]^2$$

where the effects of the forces due to buoyancy and lift are ignored. Then, the bubble diameter at detachment from the porous surface becomes

$$D^{16/7} = \frac{32 g_c \cdot d \cdot \sigma \cdot \delta^{2/7}}{C_D \cdot \rho_L \cdot u_{L\infty}^2}. \qquad (5)$$

The drag coefficient of an air bubble moving in the liquid has been measured by Peebles and Garber [3] as

$$C_D = 18{\cdot}7 \left(\frac{D\bar{u}_L}{\nu_L}\right)^{-2/3} \qquad (6)$$

Substituting (6) into (5) gives

$$D = \left(\frac{1{\cdot}252 g_c^3 d^3 \sigma^3}{\rho_L \mu_L^2 u_{L\infty}^4}\right)^{7/32} \cdot \delta^{1/8}. \qquad (7)$$

Then the boundary layer thickness δ can be determined from equations (1), (2), (3), (4) and (7). By introducing (2) and (3), and the slip ratio $\alpha \equiv u_G/u_L$, (1) can be re-written as

$$\frac{d}{dx}\int_0^\delta \left[R_G\left(1 - \alpha \frac{\rho_G}{\rho_L} \cdot \frac{u_L}{u_{L\infty}}\right)\alpha \frac{u_L}{u_{L\infty}} + (1 - R_G)\left(1 - \frac{u_L}{u_{L\infty}}\right)\frac{u_L}{u_{L\infty}}\right] dy - \frac{Q_G}{u_{L\infty}}$$
$$= \tfrac{1}{2}\left(2{\cdot}87 + 1{\cdot}58 \ln \frac{x}{D_m}\right)^{-2{\cdot}5}. \qquad (8)$$

It is difficult to assume the void distribution R_G in the boundary layer. For simplicity in the calculation, it is assumed to be given by

$$R_G = R_{G_w}\left[1 - \left(\frac{y}{\delta}\right)^n\right], \quad (n > 0) \qquad (9)$$

where R_{G_w} is the void fraction at the wall and is equated to the equivalent pore diameter d and the number of pores per unit area of the wall surface N as

$$R_{G_w} = \frac{N\pi d^2}{4}. \qquad (10)$$

From the continuity equation, this R_G is equated to the injection rate Q_G as follows:

$$\int_0^\delta R_G \frac{u_G}{u_{L\infty}} dy = \int_0^\delta R_G \alpha \frac{u_L}{u_{L\infty}} dy$$
$$= \int_0^\delta R_G \alpha \left(\frac{y}{\delta}\right)^{1/7} dy = \frac{Q_G}{u_{L\infty}}(x - x_0). \qquad (11)$$

Introducing (9) into (11) gives the range of parameters to satisfy the condition $n > 0$ as

$$0 < \frac{Q_G}{u_{L\infty}} \cdot \frac{x - x_0}{\delta} < \frac{7}{8}\alpha R_{G_w}. \qquad (12)$$

Beyond this range, R is assumed to be given by $R_G = R_{G_w} =$ constant,

$$\text{for } \frac{Q_G}{u_{L\infty}} \cdot \frac{x - x_0}{\delta} \geqslant \frac{7}{8} \alpha R_{G_w}. \quad (13)$$

Substituting (2), (7), (9), (10) and (11) into (8), and rearranging terms result in

$$\frac{dRe_\delta}{dRe_x} = \frac{\left[\frac{2}{\alpha}\frac{Q_G}{u_{L\infty}} + \left(C + 1{\cdot}58 \ln \frac{Re_x}{Re_\delta^{1/8}}\right)^{-2\cdot5}\right]\left[63\,\alpha\, R_{G_w}\frac{Re_\delta}{Re_x} - 8\frac{Q_G}{u_{L\infty}}\left(1 - \frac{Re_{x_0}}{Re_x}\right)\right]^2 - 6272\,\alpha\, R_{G_w}^2\left(1 - \alpha^2\frac{\rho_G}{\rho_L}\right)\frac{Q_G}{u_{L\infty}}\left(\frac{Re_\delta}{Re_x}\right)^2}{\left[\frac{7}{36} - \frac{112}{9}R_{G_w}\left(1 - \alpha^2\frac{\rho_G}{\rho_L}\right)\right]\left[63\,\alpha\, R_{G_w}\frac{Re_\delta}{Re_x} - 8\frac{Q_G}{u_{L\infty}}\left(1 - \frac{Re_{x_0}}{Re_x}\right)\right]^2 + 784\alpha R_{G_w}^2\left(1 - \alpha^2\frac{\rho_G}{\rho_L}\right)\frac{Re_\delta}{Re_x}\left[63\,\alpha\, R_{G_w}\frac{Re_\delta}{Re_x} - 16\frac{Q_G}{u_{L\infty}}\left(1 - \frac{Re_{x_0}}{Re_x}\right)\right]}$$

for

$$0 < \frac{Q_G}{u_{L\infty}} \cdot \frac{Re_x}{Re_\delta}\left(1 - \frac{Re_{x_0}}{Re_x}\right) < \frac{7}{8}\alpha R_{G_w} \quad (14)$$

where

$$Re_x \equiv \frac{x u_{L\infty}}{\nu_L}, \quad Re_{x_0} = \frac{x_0 u_{L\infty}}{\nu_L}, \quad Re_\delta \equiv \frac{\delta u_{L\infty}}{\nu_L} \quad (15)$$

$$C \equiv 2{\cdot}87 + 1{\cdot}58 \ln B \quad (16)$$

$$B \equiv \left(\frac{1{\cdot}44 \mu_L^2}{g_c d \sigma \rho_L}\right)^{21/32}. \quad (17)$$

For the region

$$\frac{Q_G}{u_{L\infty}} \cdot \frac{Re_x}{Re_\delta}\left(1 - \frac{Re_{x_0}}{Re_x}\right) \geqslant \frac{7}{8}\alpha R_{G_w}$$

substituting (2), (7), (10) and (13) into (8) results in the following equation:

$$\frac{dRe_\delta}{dRe_x} = \frac{\frac{2}{\alpha} \cdot \frac{Q_G}{u_{L\infty}} + \left(C + 1{\cdot}58 \ln \frac{Re_x}{Re_\delta^{1/8}}\right)^{-2\cdot5}}{\frac{7}{36} + \frac{14}{9}R_{G_w}\left(1 - \alpha^2\frac{\rho_G}{\rho_L}\right)}$$

$$\text{for } \frac{Q_G}{u_{L\infty}} \cdot \frac{Re_x}{Re_\delta}\left(1 - \frac{Re_{x_0}}{Re_x}\right) \geqslant \frac{7}{8}\alpha R_{G_w}. \quad (18)$$

The boundary layer thickness is determined by integrating (14) or (18). Then the local skin friction coefficient f_w is obtained by using those calculated values of the boundary layer thickness as

$$f_w = \frac{\tau_w}{\frac{1}{2}\rho_L u_{L\infty}^2} = \left(C + 1{\cdot}58 \ln \frac{Re_x}{Re_\delta^{1/8}}\right)^{-2\cdot5}. \quad (19)$$

The boundary layer flow treated here can be considered as highly turbulent flow by the effect of gas injection. In this case, the molecular diffusivity of momentum and heat is considered to be small enough compared with the eddy diffusivity throughout the boundary layer. If the existence of the laminar sublayer on the wall surface is ignored, the heat transfer coefficient can simply be determined from the values of the skin friction coefficient by assuming Reynolds' analogy:

$$St_x = \frac{h_x}{\rho_L c_p u_{L\infty}} = \frac{f_w}{2}. \quad (20)$$

Although the covered range is rather limited, an experimental investigation [4] for the forced flow, nucleate boiling heat transfer suggests that this analogy could be applicable for the case that the effect of the bubble generation on the friction and heat transfer is a hydrodynamic one.

RESULTS

Equivalent pore diameter d and number of the pores per unit area N

The porous material is considered here to be made of sintered small spheres, the diameter of which is denoted as d', as shown in Fig. 4. The equivalent pore diameter d can be obtained from the equation

$$\frac{\pi}{4} d^2 = d'^2\left(1 - \frac{\pi}{4}\right). \quad (21)$$

On the other hand, the number of the pores per unit area N can be written as

$$N = \frac{1}{d'^2}. \tag{22}$$

Substituting (22) into (21) gives

$$\pi N d^2 = 4 - \pi = 0{\cdot}86. \tag{23}$$

Slip ratio α

There is no available data for the slip ratio between the bubble and the liquid flowing in a horizontal tube. The slip ratio $\alpha = u_G/u_L$ is

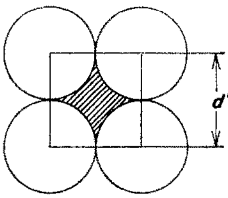

FIG. 4. Calculation of the equivalent pore diameter.

generally considered as a function of the void fraction R_G. It can be considered from the available data for a vertical tube [5] that, when the void fraction decreases to zero, the slip ratio approaches unity. For simplicity in the calculation, the slip ratio is assumed here to be unity.

Air injection into water flow

The air injection through a porous flat plate into the boundary layer of water flowing over the surface was investigated here. The temperature of the flow field is selected as 60°F. The selected parameters C, which depend on the pore diameter, are $+0{\cdot}17$ and $-1{\cdot}91$. These values correspond to the pore diameters of $d = 0{\cdot}01$ (mm) $= 3{\cdot}28 \times 10^{-5}$ (ft) and $d = 1{\cdot}0$ (mm) $= 3{\cdot}28 \times 10^{-3}$ (ft), respectively. The selected injection parameters $Q_G/u_{L\infty}$ are 0·001, 0·01, and 0·1. The numerical solutions of (14) and (18) were obtained by using the Runge–Kutta method. By using the calculated values of Re_δ for various Re_x, the values of the local skin friction coefficient f_w are obtained from (19). The calculated results of local and also total skin friction coefficient are shown in Figs. 5a and b. It was assumed here that the initial condition for the numerical integration is taken as $Re_\delta =$ 146·8 at $Re_{x_0} = 10^3$. This value was calculated from the equation

$$Re_\delta = 4{\cdot}64\sqrt{(Re_x)} \tag{24}$$

by assuming that there exists a small leading section covered with a laminar boundary layer.

2. CALCULATION ON A COMPLETELY SEPARATED TWO-PHASE BOUNDARY LAYER. ANALYSIS

In some circumstances, the gas bubbles injected from the porous plate into a liquid flowing over it coagulate and make a stable and continuous film of gas over the surface. In such a case, the heat transfer and the friction drag are considerably reduced by the insulating effect of the gas layer. Some solutions of this kind of problem are presented [6] within the framework of laminar boundary-layer theory.

The second section of the present investigation is intended to show the effects of the gas injection rate and Reynolds number of the free stream on the reduction of the skin friction drag by using more simplified theory.

The physical model is shown in Fig. 6. A stream of pure liquid approaches a flat plate. A gas is injected uniformly from the surface into the boundary layer, and makes a continuous film over the surface. It is assumed that the gas phase is non-condensing and the evaporation at the liquid–gas interface is ignored. The buoyancy effect is also neglected.

The velocity profile in the gas layer is assumed to be given by a linear equation as

$$\frac{u_G}{u_{LG}} = \frac{y}{\delta_G} \tag{25}$$

where u_{LG} is the velocity at the liquid–gas interface. The continuity equation for the gas phase is

$$Q_G x = \int_0^\delta u_G \, dy = \tfrac{1}{2} u_{LG} \delta_G \tag{26}$$

where the length of the non-porous leading section is assumed to be zero.

At the interface, the balance of the shear forces of both phases exists as

$$\mu_G \frac{u_{LG}}{\delta_G} = \mu_L \left(\frac{\partial u_L}{\partial y}\right)_{y=} \quad . \tag{27}$$

If it is assumed that the thickness of gas layer

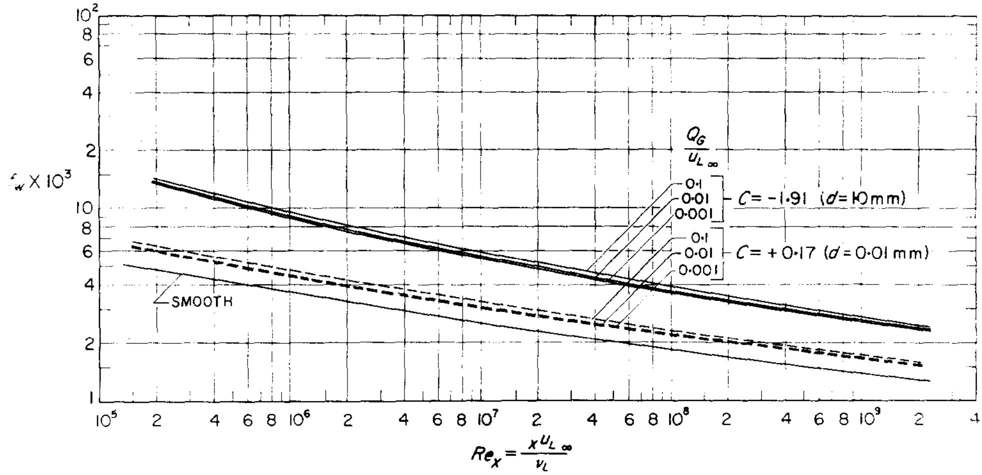

FIG. 5a. Coefficient of local skin friction of water flow on porous flat plate with air injection.

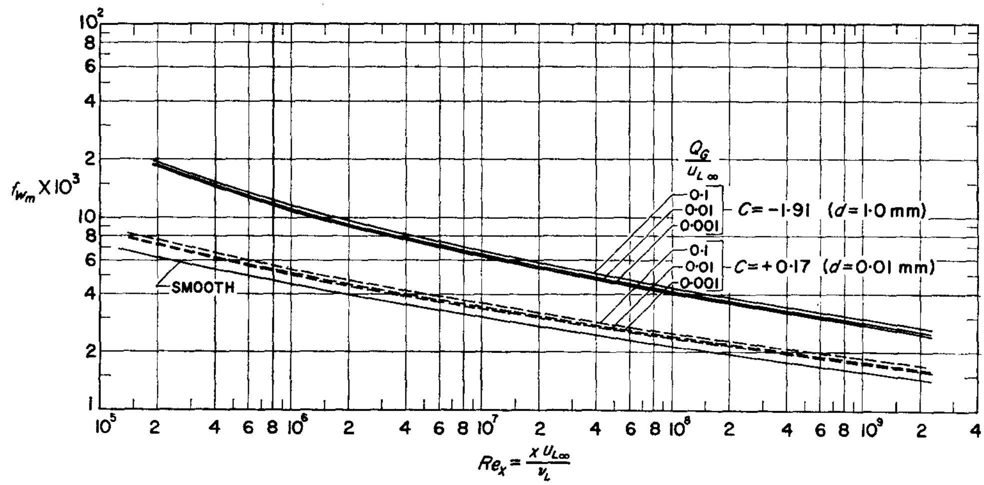

FIG. 5b. Coefficient of total skin friction of water flow on porous flat plate with air injection.

increases slowly with x, the shear force of the liquid phase at the interface can be approximated by the shear force on a smooth flat plate which is moving with the liquid in the same direction at an interface velocity. Then (27) becomes, for a case in which the liquid boundary layer is laminar,

$$\mu_G \frac{u_{LG}}{\delta_G} = \tfrac{1}{2}\rho_L(u_{L\infty} - u_{LG})^2 \frac{0.664}{\left[\frac{(u_{L\infty} - u_{LG})x}{\nu_L}\right]^{0.5}} \tag{28}$$

by assuming Blasius' equation:

$$f_{w_0} = \frac{0.664}{Re_x^{0.5}} \tag{29}$$

and for a case in which the liquid boundary layer is turbulent,

$$\mu_G \frac{u_{LG}}{\mu_G} = \tfrac{1}{2}\rho_L(u_{L\infty} - u_{LG})^2 \frac{0.0592}{\left[\frac{(u_{L\infty} - u_{LG})x}{\nu_L}\right]^{0.2}} \tag{30}$$

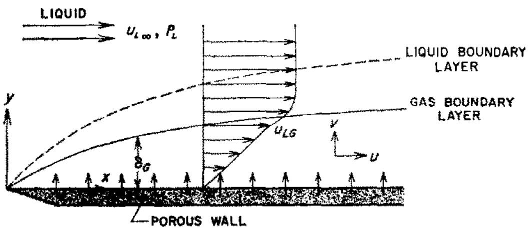

FIG. 6. Physical model and co-ordinate system.

by assuming Prandtl's equation:

$$f_{w_0} = \frac{0\cdot0592}{Re_x^{0\cdot2}} \quad (31)$$

where f_{w_0} denotes the local skin friction coefficient on a smooth flat plate without gas injection. Rearranging the terms gives

$$\frac{\xi^2}{(1-\xi)^{1\cdot5}} = 0\cdot664 \frac{\mu_L}{\mu_G} \cdot \frac{Q_G}{u_{L\infty}} \cdot Re_x^{0\cdot5} \quad \text{(laminar)} \quad (32)$$

$$\frac{\xi^2}{(1-\xi)^{1\cdot8}} = 0\cdot0592 \frac{\mu_L}{\mu_G} \cdot \frac{Q_G}{u_{L\infty}} \cdot Re_x^{0\cdot8} \quad \text{(turbulent)} \quad (33)$$

where $\xi = \frac{u_{LG}}{u_{L\infty}}$.

The skin friction coefficient with gas injection can be obtained from

$$f_w = \frac{\tau_w}{\frac{1}{2}\rho_L u_{L\infty}^2} = \frac{1}{\frac{1}{2}\rho_L u_{L\infty}^2} \cdot \left(\mu_G \frac{u_{LG}}{\delta_G}\right). \quad (34)$$

Introducing (28) and (30) into (34) gives

$$\frac{f_w}{f_{w_0}} = (1-\xi)^{1\cdot5} \quad \text{(laminar)} \quad (35)$$

$$= (1-\xi)^{1\cdot8} \quad \text{(turbulent)}. \quad (36)$$

The velocity ratio $\xi = u_{LG}/u_{L\infty}$ is calculated from (32) or (33) for various values of $Q_G/u_{L\infty}$ and Re_x. Using the calculated values of ξ, the local skin friction coefficient f_w can be obtained from (35) or (36).

Assuming the temperature profile in the gas layer to be linear, the heat balance at the liquid–gas interface is

$$q_x = \frac{k_L}{x} Nu_{x_0}(T_{L\infty} - T_{LG}) = \frac{k_G}{\delta_G}(T_{LG} - T_w) \quad (37)$$

where Nu_{x_0} denotes the local Nusselt number on a flat plate without gas injection and it is obtained from text books as

$$Nu_{x_0} = 0\cdot332\, Pr_L^{1/3} Re_x^{0\cdot5} \quad (38)$$

for the case of laminar liquid boundary layer, or

$$Nu_{x_0} = 0\cdot0296\, Pr_L^{1/3} Re_x^{0\cdot8} \quad (39)$$

for the case of turbulent liquid boundary layer.

If the heat transfer coefficient with gas injection is defined as

$$q_x = h_x(T_{L\infty} - T_w) \quad (40)$$

introducing (40) and the velocity ratio ξ into (37) gives

$$\frac{Nu_x}{Nu_{x_0}} = \frac{1}{1 + \frac{2Q_G}{u_{L\infty}} \frac{1}{\xi} \frac{k_L}{k_G} Nu_{x_0}} \quad (41)$$

where Nu_x is the local Nusselt number with gas injection defined as

$$Nu_x \equiv \frac{h_x x}{k_L}. \quad (42)$$

The heat transfer coefficient with gas injection can be calculated from (41), together with (32) and (38) for the case of laminar liquid boundary layer, or with (33) and (39) for the case of turbulent liquid boundary layer.

RESULTS

The uniform air injection through a porous flat plate into the boundary layer of water flowing over the surface was investigated. The selected temperature of the flow field is 68°F. Some calculated results for the skin friction coefficient are shown in Fig. 7. A significant

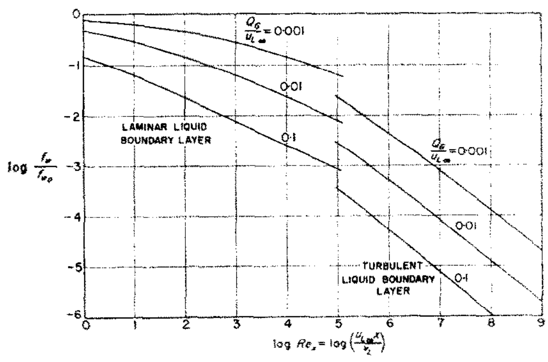

FIG. 7. Reduction of the local skin friction coefficient.

decrease in the skin friction coefficient can be expected as shown in Fig. 7 by producing a stable and continuous gas layer between a surface and a liquid flowing over it.

CONCLUSIONS

The effect of gas injection through a porous flat plate on the skin friction and heat transfer characteristics in the boundary layer of a liquid flowing over the surface is studied.

The calculated results for a case of bubbling by gas injection show a significant increase in the skin friction and the heat transfer coefficient. The porosity of the wall has a considerable effect on the increase in the skin friction and the heat transfer; on the other hand, the effect of changing the rate of gas injection is less significant.

Calculations for a case in which the injected gas makes a continuous film over the surface show a significant decrease in the skin friction and the heat transfer coefficient.

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Résumé—Quelques calculs théoriques ont été faits pour déterminer l'effet du bouillonnement produit, par l'injection de gaz à travers une paroi poreuse, sur le frottement à la paroi et les échanges thermiques dans la couche limite d'un liquide s'écoulant sur cette paroi. Les résultats montrent un accroissement sensible à la fois du coefficient de frottement pariétal et du coefficient d'échange thermique.

Des calculs effectués dans le cas où le gaz injecté fait un film continu à la surface sont également présentés. Dans ce cas, le frottement à la paroi et la transmission de chaleur décroissent sérieusement.

Zusammenfassung—Der Einfluss der Rührwirkung eines durch eine poröse Wand eingeblasenen Gases auf Wandreibung und Wärmeübergang in der Grenzschicht einer strömenden Flüssigkeit wurde mit Hilfe einiger theoretischer Rechnungen bestimmt. Die Ergebnisse zeigen ein deutliches Ansteigen sowohl der Wandreibung als auch des Wärmeübergangskoeffizienten. Einige Rechnungen sind für den Fall durchgeführt, dass das eingeblasene Gas an der Wand einen zusammenhängenden Film bildet. Dafür nehmen Wandreibung und Wärmeübergang sehr stark ab.

Аннотация—Были выполнены некоторые теоретические расчёты для определения влияния пузырьков. образованных при вдуве газа через пористую стенку, на характеристики поверхностного трения и теплообмена в пограничном слое жидкости, омывающей стенку. Результаты вычисления указывают на значительное увеличение как поверхностного трения, так и коэффициента теплообмена.

Приведены также некоторые расчёты для, случая когда вдуваемый газ образует у поверхности непрерывную плёнку. В этом слунае поверхностное трение и теплообмен резко уменьшаются.